an electron charge and m is the charge state of an ion. Therefore, better propellants must have lower ionization potentials, allowing maximum m. The gain in specific impulse I_{sp} will be proportional to $m^{1/2}$. From this point of view, the proposed use of onboard hydrogen as a second stage, after exhaustion of the airbreathing mode,[4] will hardly have any advantage over any other similar propellant. On the contrary, solid propellants with low ionization potentials, such as lead or perhaps vanadium, may provide a high I_{sp}.

Figure 1 shows the dependence of specific impulse on the atomic mass of a target, irradiated by 0.1-ps-long pulses from a Ti:sapphire laser/regenerative amplifier system. The data were deduced from time-of-flight measurements of ion velocity and yield vs angle to the target surface and are described in detail elsewhere.[17]

Absolute values of I_{sp} are approaching 3×10^3 s, and, excluding lead, I_{sp} is roughly inversely proportional to the square root of mass (i.e., I_{sp} is strongly material dependent). For the majority of ablated elements, the kinetic energy E_K of individual ions varied within a relatively narrow range of ~20–50 eV. When kinetic energy is roughly constant, the speed is inversely proportional to the square root of mass and so is the specific impulse. Lead ions exhibited very high mean $E_K = 571$ eV. This effect, at least partly, can be caused by multiple ionization.[17]

Conclusion

To conclude this brief Note, it seems that ALP based on picosecond or subpicosecond pulses can be a reasonable alternative to existing LP concepts. The approach simplifies the construction of LP-driven vehicles and can resolve other limitations endangering the progress of LP using LSC/LSD wave mediated momentum transfer. Preliminary data indicate that ALP will have its own place in future space transportation applications.

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Criterion to Determine the Number of Modes in a Frequency Band

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Introduction

IN the time domain, selecting the model order, or the number of modes in a frequency band, is a key first step toward the goal of estimating the modal parameters of a vibrating system. Several information theoretic criteria have been proposed for this model-order selection task. Akaike has provided two criteria. His first criterion is the final prediction error (FPE)[1] criterion and selects the order so that the average error variance for a one-step prediction is minimized. Akaike also suggested another selection criterion using a maximum likelihood approach to derive a criterion termed the Akaike information criterion (AIC).[2] The AIC determines the model order by minimizing an information theoretic function. FPE and AIC are asymptotically equivalent but do not yield consistent estimates of the model order; the result is a tendency to overestimate the order as the data record length increases.[3] In response to this, another effective criterion, the minimum description length (MDL) criterion, is proposed by Schwarz[4] and Rissanen.[5] This criterion uses a penalty function, which provides consistent estimation of the model order. All of these methods are applicable only to scalar processes, and a generalization to multivariate processes remains to be established.

In this Note, using a combination of an overdetermined instrumental variable scheme[6] and the multivariate MDL criterion, we propose a new method for autoregressive (AR) order determination of a vector-autoregressive moving average or VARMA(p, q) process. To determine p, the order of the multivariate AR part, an overdetermined instrumental variable product moment matrix is defined. Once p has been estimated, the number of modes in a frequency band is derived. In a mechanical structure the number n of modes in a frequency band is related to the order p of the AR part and to the number of sensors m by the relation $p = 2n/m$.

The applicability of the proposed procedure to typical engineering problems is the determination of the model order from accelerometer output only, which is the first step in the estimation of modal parameters. The second step is the identification of these modal parameters using the eigensystem realization algorithm[7] or AR coefficients[8,9] of a VARMA model. The proposed method is experimentally applied to acceleration signals. But this method is easily generalized to other systems that use displacement or velocities and more generally an observation data vector, which has a VARMA representation.

Received 3 May 1999; revision received 30 September 1999; accepted for publication 1 October 1999.

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VARMA Representation

We consider a structure excited by an unknown random Gaussian force. The model used to represent the dynamics of this mechanical system is the mixed VARMA (or multivariate) process[8]:

$$y_t = \sum_{i=1}^{p} A_i y_{t-i} + \sum_{j=0}^{q} B_j u_{t-j} \tag{1}$$

where y_t is the discrete observation vector obtained from m accelerometers and u_t is a white Gaussian noise. This equation contains the AR part with AR matrix coefficients A_i and the moving average (MA) part with MA matrix coefficients B_j. The order of the AR part is p, and the order of the MA part is q. Because the true orders p and q are unknown, we consider the general case of Eq. (1) with (p, q) replaced by unknown (p', q'). We develop Eq. (1) for $t = 0, \ldots, N-1$ and form the system

$$[[A_0] - [A_1] - \cdots - [A_{p'}]] \times \begin{bmatrix} y_0 & y_1 & y_2 & \cdot & y_{p'} & y_{p'+1} & \cdot & y_{N-1} \\ 0 & y_0 & y_1 & \cdot & y_{p'-1} & y_{p'} & \cdot & y_{N-2} \\ 0 & 0 & y_0 & \cdot & y_{p'-2} & y_{p'-1} & \cdot & y_{N-3} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & y_0 & y_1 & \cdot & y_{N-1-p'} \end{bmatrix}$$
$$= [[B_0][B_1]\cdots[B_{q'}]] \times \begin{bmatrix} u_0 & u_1 & u_2 & \cdot & u_{p'} & u_{p'+1} & \cdot & u_{N-1} \\ 0 & u_0 & u_1 & \cdot & u_{p'-1} & u_{p'} & \cdot & u_{N-2} \\ 0 & 0 & u_0 & \cdot & u_{p'-2} & u_{p'-1} & \cdot & u_{N-3} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & u_{p'-q'} & u_{p'-q'+1} & \cdot & u_{N-1-q'} \end{bmatrix} \tag{2}$$

with N the data length and $[A_0] = [I_m]$.

We can write the preceding system as

$$\Psi_{p'} Y_{p'} = \Omega_{p'} \tag{3}$$

with $Y_{p'}$ the $m(p'+1) \times N$ matrix of data, $\Omega_{p'}$ the $m \times N$ matrix of coefficients MA and white Gaussian noise, and u_t and $\Psi_{p'}$ the $m \times m(p'+1)$ matrix that contains the AR parameters:

$$\Psi_{p'} = \lfloor A_0 - A_1 - \cdots - A_{p'} \rfloor \tag{4}$$

The multivariate minimum description length criterion is now developed to determine the number of AR coefficients from the $\{y_t\}$ process.

Determination of the Number of Modes in a Frequency Band by a Multivariate MDL Criterion

To develop a multivariate MDL criterion, we introduce an extended instrumental variable[6] matrix Z_k of dimension $m(k+1) \times N$:

$$Z_k = \begin{bmatrix} z_0 & z_1 & \cdots & z_k & \cdots & z_{N-1} \\ 0 & z_0 & \cdots & z_{k-1} & \cdots & z_{N-2} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & z_0 & \cdots & z_{N-k-1} \end{bmatrix} \tag{5}$$

in which z_i is a vectorial instrumental variable sequence. The instrumental variable sequence is highly correlated with the observed sequence y_i but completely uncorrelated with the noise.[6] The dimension of $Z_k Y_{p'}^T$ is $m(k+1) \times m(p'+1)$, and increasing the number of rows in $Z_k Y_{p'}^T$ implies the use of more information. An overdetermined instrumental variable method is then preferable. As such, we take $k > p'$. Premultiplying Eq. (3) by $(1/N)Z_k^T$, we obtain

$$(1/N)\Psi_{p'} Y_{p'} Z_k^T = (1/N)\Omega_{p'} Z_k^T \tag{6}$$

Putting $W_{p'} = (1/N) Y_{p'} Z_k^T$ matrix $m(p'+1) \times m(k+1)$ and $V_k = (1/N)\, \Omega_{p'} Z_k^T$ matrix $m \times m(k+1)$, we obtain

$$\Psi_{p'} W_{p'} = V_k \tag{7}$$

Furthermore, we define the $m(p'+1) \times m(p'+1)$ overdetermined instrumental variable product moment matrix

$$\Gamma_{p'} = W_{p'} W_{p'}^T = (1/N^2) Y_{p'} Z_k^T Z_k Y_{p'}^T \tag{8}$$

Note that $\Gamma_{p'}$ is a symmetric positive semidefinite matrix, and we consider both the matrix $\Gamma_{p'}$ and the multivariate MDL criterion to obtain the order p of the AR part of the VARMA process. Following Schwarz[4] and Rissanen,[5] the MDL criterion is equal to the sum of the log-likelihood function of the maximum likelihood estimator of the model parameters and a function that penalizes the use of a large number of model parameters. In the multivariate case the number of free-adjusted parameters in the AR part is $m^2(p'+1)$. Denote $V_k = [v_1, v_2, \ldots, v_{m(k+1)}]$. This matrix consists of $m(k+1)$ independent, m-dimensional, normal random vectors v_i, with zero mean and unknown covariance matrix $Q_v = E[v_i \ v_i^T]$, and thus the MDL criterion is given by

$$J_{\mathrm{MDL}}(p') = -\log f\left[v_1, \ldots, v_{m(k+1)}\right] + \tfrac{1}{2} m^2 (p'+1) \log[m(k+1)] \tag{9}$$

where $f[v_1, \ldots, v_{m(k+1)}]$ denotes the probability density function

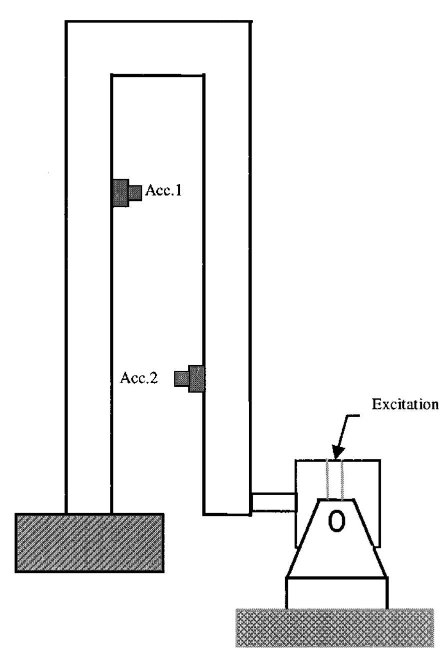

Fig. 1 Experimental U beam.

Table 1 Statistics of $J(p')$ with $h = 1$, $N =$ 16,384, and two accelerometers

k	$p' = 1$	$p' = 2$	$p' = 3$	$p' = 4$	$p' = 5$	$p' = 6$	$p' = 7$	$p' = 8$	$p' = 9$	$p' = 10$	$p' = 11$
20	3.2×10^6	2.1×10^6	1.5×10^6	4.9×10^5	4.1×10^5	5.1×10^2	1.5×10^2	1.6×10^2	10^2	60	63
30	5×10^6	2.7×10^6	1.9×10^6	6.3×10^5	5.1×10^5	6.6×10^2	1.2×10^2	1.3×10^2	84	45	46
40	8.3×10^6	3.3×10^6	2.3×10^6	7.8×10^5	5.8×10^5	6.9×10^2	1.1×10^2	1.2×10^2	76	37	37

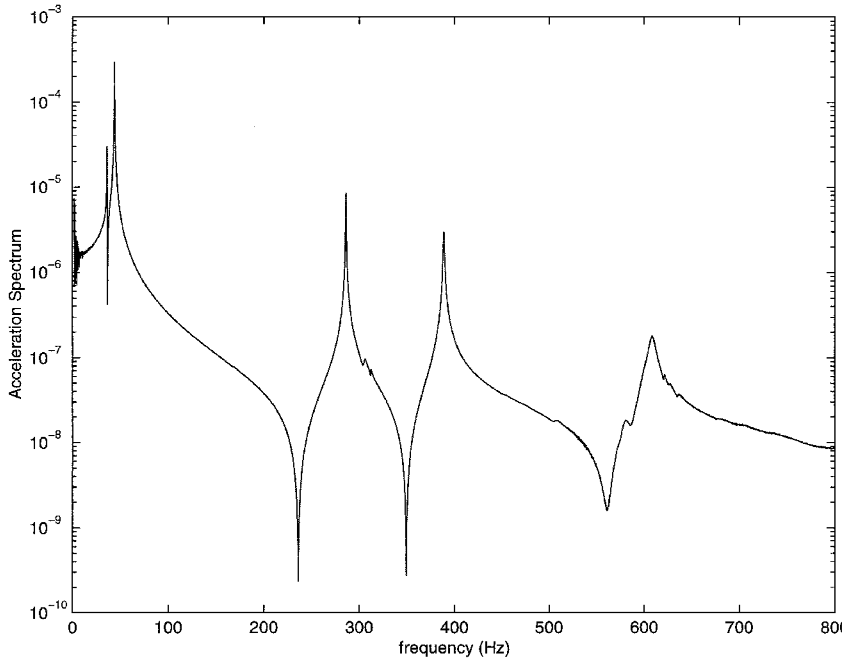

Fig. 2 Frequency response of channel 1.

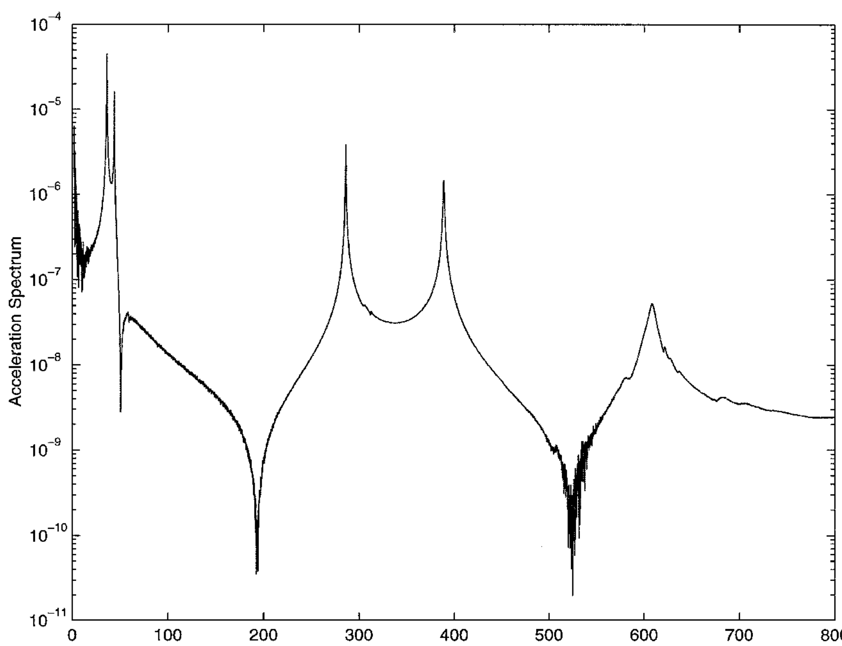

Fig. 3 Frequency response of channel 2.

of $\{v_i\}$. For a multivariate normal model we have

$$J_{\mathrm{MDL}}(p') = m^2[(k+1)/2]\log(2\pi) + m[(k+1)/2]\log[\det(Q_v)] + \tfrac{1}{2}\mathrm{tr}\left(Q_v^{-1}\Psi_{p'}\Gamma_{p'}\Psi_{p'}^T\right) + \tfrac{1}{2}m^2(p'+1)\log[m(k+1)] \quad (10)$$

For fixed p' the matrix Q_v that minimizes the criterion (10) is $Q_v = \Psi_{p'}\Gamma_{p'}\Psi_{p'}^T$ and we obtain the minimum value of $\det(Q_v)$ by the eigenvalue decomposition of $\Gamma_{p'}$:

$$\Gamma_{p'} = [S_\Lambda \quad S_\varepsilon]\begin{pmatrix}\Lambda & 0\\ 0 & \varepsilon\end{pmatrix}\begin{pmatrix}S_\Lambda^T\\ S_\varepsilon^T\end{pmatrix} \quad (11)$$

where Λ is the diagonal matrix containing the mp' largest eigenvalues in decreasing order and the columns of S_Λ are the corresponding eigenvectors. ε is the diagonal matrix that contains the m smallest eigenvalues, and the columns of S_ε are the corresponding eigenvectors. Constraining $\Psi_{p'}$ to be orthogonal, the choice of $\Psi_{p'}$ that minimizes the criterion (10) is found to be the matrix of eigenvectors associated with the minimum eigenvalues of $\Gamma_{p'}$. With this choice we have $\Psi_{p'} = S_\varepsilon^T$, and we obtain

$$Q_v = S_\varepsilon^T\left[S_\Lambda \Lambda S_\Lambda^T + S_\varepsilon \varepsilon S_\varepsilon^T\right]S_\varepsilon \quad (12)$$

Because the eigenvectors are orthonormal, we obtain $Q_v = \varepsilon$ and

$$\det(Q_v) = \prod_{i=1}^{m}\varepsilon_i$$

where ε_i are the smallest eigenvalues of $\Gamma_{p'}$. Substituting into Eq. (10) and dropping all terms that do not depend on p' or $\Psi_{p'}$, we obtain

$$J_{\mathrm{MDL}}(p')\frac{2}{m(k+1)} = \log\left[\prod_{i=1}^{m}\varepsilon_i m(k+1)^{mp'/(k+1)}\right] \quad (13)$$

Because the function log(·) is a monotonically increasing function, we can form a different criterion that contains exactly the same information as $J_{\mathrm{MDL}}(p')$. The new criterion chosen is

$$J(p') = \left(\prod_{i=1}^{m} \varepsilon_i\right) m(k+1)^{mp'/(k+1)} \tag{14}$$

There are several ways to choose the instruments and the matrix Z_k. In this Note we consider only the special case where $z_t = y_{t-h}$, and for the selection of instruments we consider different values of h. We can then form the new multivariate minimum description length criterion (14) and search for an abrupt change in this criterion for different values of p'.

Experimental Results

The procedure for multivariate AR-order determination, or for the estimation of the number of modes in a frequency band, is now applied in the laboratory to a real structure. In this experience we consider a U beam with two accelerometers (Fig. 1). This beam is randomly excited. The signals are sampled at the rate $\Delta t = 625\,\mu$s, and 16,384 points are collected for each channel.

Our purpose is to determine the number of modes in the frequency band [0; 800 Hz] from output accelerometers only. Figures 2 and 3 show the frequency response of accelerometers. From these figures it is very difficult to determine the number of modes of this beam in the frequency band considered by counting the number of peaks of resonance, the number of apparent peaks and very weak peaks varying between 5 and 8.

These figures do not allow us to know the exact number of modes in the frequency band of interest. To resolve this problem, we use the $J(p')$ criterion. For the selection of the instrumental variables $z_t = y_{t-h}$, we have considered different values of h and found that all of these selections could give the satisfactory results. Table 1 shows the statistical results of the criterion $J(p')$ when the number of block rows in the matrix of instruments Z_k changes. In these experimental results we use the instrumental variable $z_t = y_{t-1}$. The true order p of the AR part of the multivariate ARMA process is obtained by inspection of the values of $J(p')$: when a break occurs, we determine the order of the AR part.

By inspection of Table 1, the true order of the AR part is $p = 6$, and the number of modes in the frequency band considered is $n = 6$.

Conclusions

A time-domain procedure for the determination of the number of modes in a frequency band, from the time response delivered by the output of accelerometers only, has been proposed. Based on a combination of the multivariate minimum description length and the overdetermined instrumental variable scheme, an efficient method for AR-order determination of a multivariate ARMA model has been developed. Experimental results have shown the effectiveness of our method. It may be interesting to study other instrumental variable selections and to generalize this method to large industrial structures at work.

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Correcting System Matrices Using the Orthogonality Conditions of Distinct Measured Modes

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Nomenclature

$[K]$	= actual stiffness matrix
$[K_o]$	= analytical stiffness matrix
$[M]$	= actual mass matrix
$[M_o]$	= analytical mass matrix
$\boldsymbol{m}_{\mathrm{actual}}$	= vector of actual masses
$\boldsymbol{m}_{\mathrm{analytical}}$	= vector of analytical masses
$\boldsymbol{m}_{\mathrm{update}}$	= vector of updated masses
N	= degrees of freedom of the analytical model
N_e	= number of measured modes
$\boldsymbol{x}_j$	= jth measured mode shape
$(x_i)_r$	= rth element of $\boldsymbol{x}_i$
$\delta\boldsymbol{k}$	= vector of stiffness corrections
$[\delta K]$	= correction stiffness matrix
$[\delta M]$	= correction mass matrix
$\delta\boldsymbol{m}$	= vector of mass corrections
δm_{rs}	= (r, s)th element of $[\delta M]$
ϵ_k	= error parameter for the updated stiffnesses
ϵ_m	= error parameter for the updated masses
ϵ_λ	= error parameter for the updated eigenvalues
$(\lambda'_j, \boldsymbol{x}'_j)$	= jth mode of vibration of the mass-modified updated system
$(\lambda''_j, \boldsymbol{x}''_j)$	= jth mode of vibration of the mass-modified actual system

Introduction

WITH the arrival of digital computers, new methods of analysis have been developed to analyze and predict the dynamical behavior of complex structures, especially in the method of finite elements. Once the finite element model of a physical system is constructed, it is often validated by comparing its analytical modes of vibration with the results of a modal survey. If the agreement between the two is good, then the analytical model can be used with confidence for future analysis. If the correlation between the two is poor, then assuming the measured data to be exact, the finite element model must be corrected or updated such that the correlation between analytical predictions and test data is improved. Many model updating schemes have been developed over the years to adjust the analytical finite element models using test data. Detailed discussion of every approach is beyond the scope of this Note, and interested readers are referred to the recent survey paper by Mottershead and Friswell.[1] In this Technical Note, new model updating schemes to adjust the system mass and stiffness matrices are developed.

Proposed Model Updating Algorithms

Like the perturbation model updating approach introduced in Ref. 2, the orthogonality constraints will also be used in this Note to update the system matrices. The proposed schemes, however, are

Received 22 May 1999; revision received 22 October 1999; accepted for publication 8 November 1999.

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